Zadanie 1

$$\omega_p = 2\ \frac{rad}{s}$$

$$\omega_s = 5\ \frac{rad}{s}$$

$$\alpha_p = 7\ dB$$

$$\alpha_s = 25\ dB$$

$$\Omega_p = 1$$

$$\Omega_s = \frac{\omega_s}{\omega_p} = \frac{5}{2}$$

$$K_s = \frac{\Omega_p}{\Omega_s} = \frac{2}{5}$$

$$K_d = \sqrt{\frac{10^{\frac{\alpha_p}{10}} - 1}{10^{\frac{\alpha_s}{10}} - 1}} = \sqrt{\frac{10^{0.7} - 1}{10^{2.5} - 1}} = 0.113$$

$$n \geqslant \frac{|\log K_d|}{|\log K_s|} = \frac{0.947}{0.398} = 2.38$$

$$n = 3$$

n - nieparzyste

$$A(s) = (s+1)\prod_{k=1}^{\frac{n-1}{2}}\left(s^2 + 2s\cos\left(\frac{k}{n}\pi\right) + 1\right)$$

$$A(s) = (s+1)(s^2+s+1) = s^3 + 2s^2 + 2s + 1$$

$$H(s) = \frac{1}{A(s)} = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

$$s \leftarrow s\sqrt[2n]{10^{\frac{\alpha_p}{10}} - 1} \Rightarrow s \leftarrow s\sqrt[6]{10^{0.7} - 1} \Rightarrow s \leftarrow s \cdot 1.261$$

$$H(s) = \frac{1}{2.005s^3 + 3.180s^2 + 2.522s + 1}$$

$$s \leftarrow \frac{s}{\omega_p} \Rightarrow s \leftarrow \frac{s}{2}$$

$$H(s) = \frac{1}{0.250s^3 + 0.795s^2 + 1.261s + 1}$$

$$H(s) = \frac{4}{s^3 + 3.18s^2 + 5.044s + 4}$$

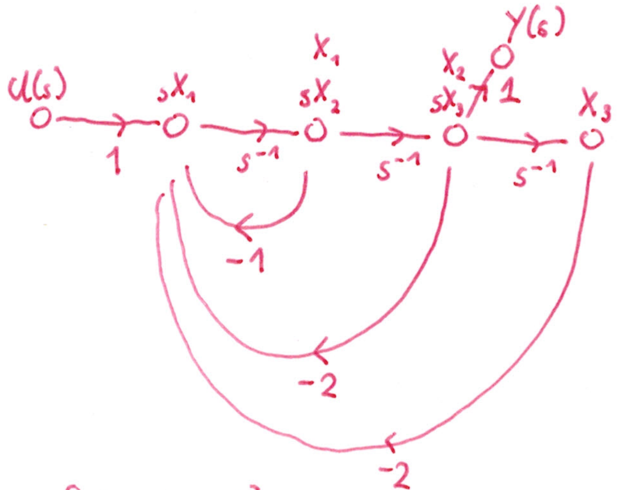
$$H(s) = \frac{s}{s^3+s^2+2s+2}$$

Zadanie 2

$$H(s) = \frac{s}{s^3+s^2+2s+2} \cdot \frac{s^{-3}}{s^{-3}} = \frac{s^{-2}}{1+s^{-1}+2s^{-2}+2s^{-3}}$$

Metoda bezpośrednia pierwsza

$$H(s) = \frac{\sum_i H_i \Delta_i}{\Delta} \qquad H(s) = \frac{s^{-2} \cdot 1}{1-(-s^{-1}-2s^{-2}-2s^{-3})}$$



$$sX_1 = -X_1 - 2X_2 - 2X_3 + 1 \cdot U$$
$$sX_2 = X_1$$
$$sX_3 = X_2$$
$$Y = 1 \cdot X_2$$

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [U]$$

$$[Y] = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + [0] \cdot [U]$$

$$\mathbb{A} = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \mathbb{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{C} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad \mathbb{D} = [0]$$

$$H(s) = \mathbb{C} \cdot (s\mathbb{I} - \mathbb{A})^{-1} \cdot \mathbb{B} + \mathbb{D}$$

$$M^{-1} = \frac{1}{\mathrm{Det}\, M} \cdot [M^D]$$

$$(s\mathbb{I} - A) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & -2 & -2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} s+1 & 2 & 2 \\ -1 & s & 0 \\ 0 & -1 & s \end{bmatrix}$$

$$\mathrm{Det}[s\mathbb{I} - A] = s^2(s+1) + 2 + 2s = s^3+s^2+2s+2$$

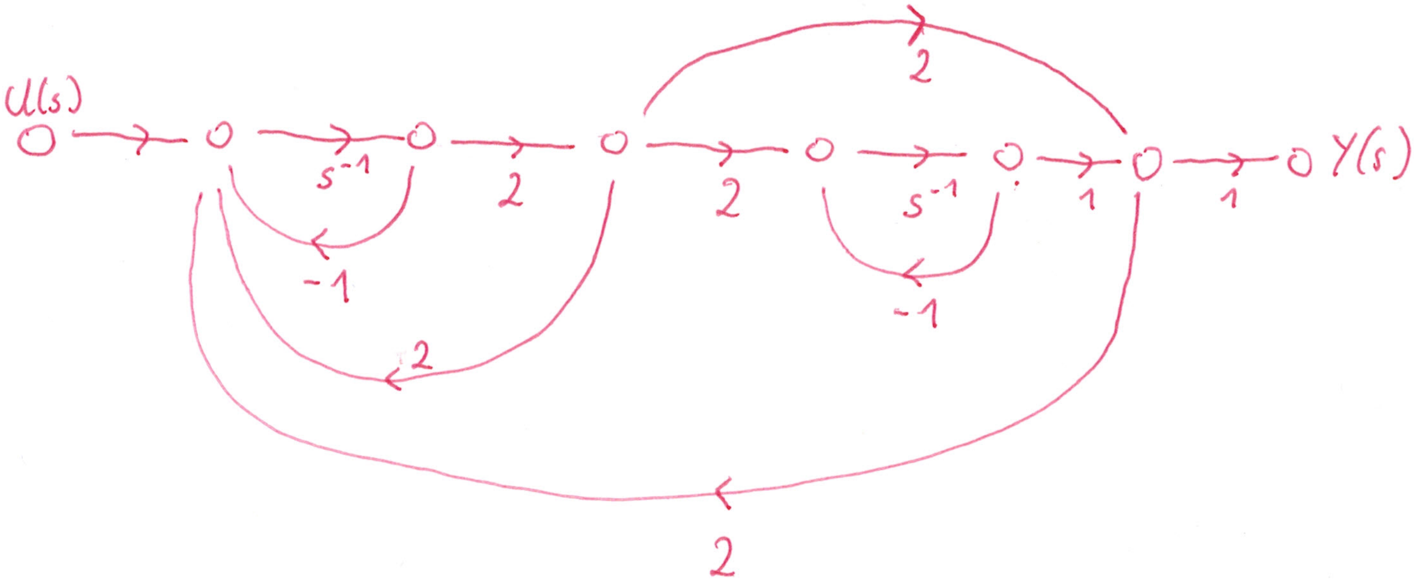
$$[s\mathbb{I} - A]^D = \begin{bmatrix} \begin{vmatrix} s & 0 \\ -1 & s \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ 0 & s \end{vmatrix} & \begin{vmatrix} -1 & s \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 2 \\ -1 & s \end{vmatrix} & \begin{vmatrix} s+1 & 2 \\ 0 & s \end{vmatrix} & -\begin{vmatrix} s+1 & 2 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 2 & 2 \\ -1 & s \end{vmatrix} & -\begin{vmatrix} s+1 & 2 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} s+1 & 2 \\ -1 & s \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} s^2 & s & 1 \\ -2s-2 & s^2+s & s+1 \\ 2s+2 & -s-3 & s^2+s+2 \end{bmatrix}^T = \begin{bmatrix} s^2 & -2s-2 & 2s+2 \\ s & s^2+s & -s-3 \\ 1 & s+1 & s^2+s+2 \end{bmatrix}$$

$$\frac{\mathbb{C}\cdot[s\mathbb{I}-A]^{D}\cdot\mathbb{B}}{\mathrm{Det}[s\mathbb{I}-A]}$$

$$\begin{array}{ccc|ccc|c}
 & & & s^2 & -2s-2 & 2s+2 & 1 \\
 & & & s & s^2+s & -s-3 & 0 \\
 & & & 1 & s+1 & s^2+s+2 & 0 \\
\hline
0 & 1 & 0 & s & s^2+s & -s-3 & s
\end{array}$$

$$H(s) = \frac{s}{s^3+s^2+2s+2}$$

Zadanie 3



$$H(s) = \frac{\sum_i H_i \Delta_i}{\Delta}$$

$$\Delta = 1 - \left(-1s^{-1} + 2\cdot(+2)\cdot s^{-1} - 1\cdot s^{-1} + 2\cdot 2\cdot 2\cdot s^{-2} + 2\cdot 2\cdot 2\cdot s^{-1}\right)$$

$$+ (-1)\cdot s^{-1}\cdot(-1)\cdot s^{-1} + s^{-1}\cdot 2\cdot(+2)\cdot s^{-1}\cdot(-1) + s^{-1}\cdot 2\cdot 2\cdot 2\cdot s^{-1}\cdot(-1) =$$

$$= 1 + s^{-1} - 4s^{-1} + s^{-1} - 8s^{-2} - 8s^{-1} + s^{-2} - 4s^{-2} - 8s^{-2} =$$

$$= 1 - 10s^{-1} - 19s^{-2}$$

$H_1 = 4s^{-1}$

$\Delta_1 = 1 + s^{-1}$

$$H(s) = \frac{4s^{-1}(1+s^{-1}) + 4s^{-2}}{1 - 10s^{-1} - 19s^{-2}} \cdot \frac{s^2}{s^2} = \frac{4s+8}{s^2 - 10s - 19}$$

$H_2 = 4s^{-2}$

$\Delta_2 = 1$

$s^2 - 10s - 19 = 0$

$\Delta_s = 100 - 4\cdot 19 = 24$

$\sqrt{\Delta_s} = 2\sqrt{6}$

$s_1 = \frac{10 - 2\sqrt{6}}{2} = 5 - \sqrt{6}$

$s_2 = \frac{10 + 2\sqrt{6}}{2} = 5 + \sqrt{6}$

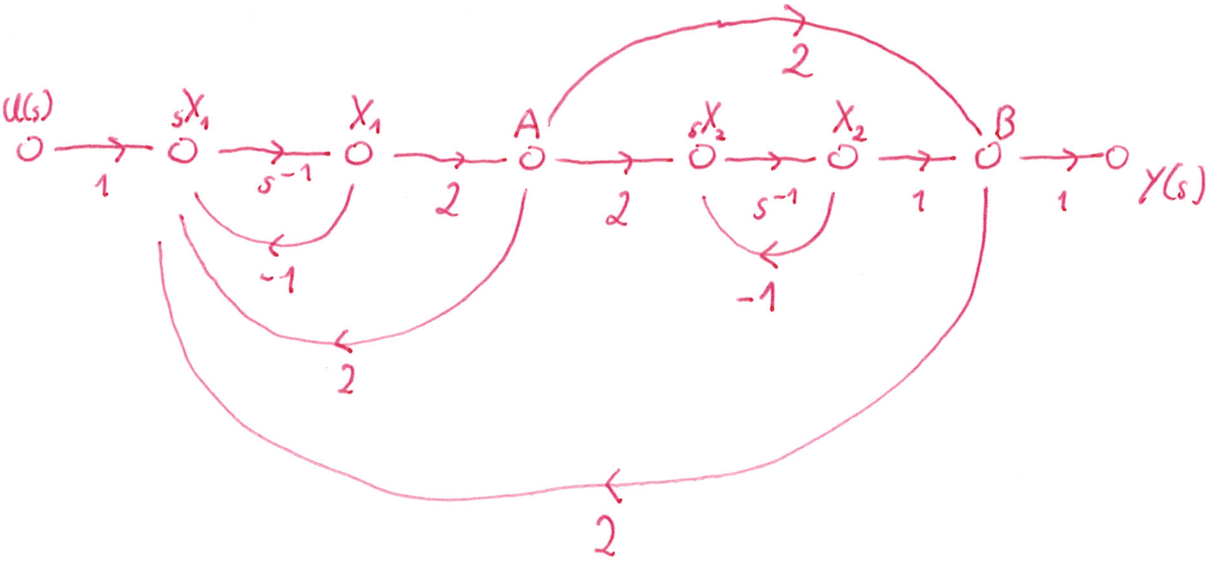
Warunek stabilności w sensie BIBO

$Re(s_1) < 0$

$Re(s_2) < 0$

Układ jest nie stabilny.

Zadanie 3



$$sX_1 = -X_1 + 2A + 2B + 1U$$
$$A = 2X_1$$
$$sX_2 = 2A - X_2$$
$$B = X_2 + 2A$$
$$Y = B$$

$$\mathbb{A} = \begin{bmatrix} 11 & 2 \\ 4 & -1 \end{bmatrix} \qquad \mathbb{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\mathbb{C} = \begin{bmatrix} 4 & 1 \end{bmatrix} \qquad \mathbb{D} = \begin{bmatrix} 0 \end{bmatrix}$$

$$H(s) = \mathbb{C} \cdot (s\mathbb{I} - \mathbb{A})^{-1} \cdot \mathbb{B} + \mathbb{D}$$

$$(s\mathbb{I} - \mathbb{A})^{-1} = \frac{1}{Det(s\mathbb{I} - \mathbb{A})} [s\mathbb{I} - \mathbb{A}]^{D}$$

$$(s\mathbb{I} - \mathbb{A}) = \begin{bmatrix} s-11 & -2 \\ -4 & s+1 \end{bmatrix}$$

$$Det(s\mathbb{I} - \mathbb{A}) = (s-11)(s+1) - 8 = s^2 - 10s - 19$$

$$sX_1 = -X_1 + 2 \cdot 2X_1 + 2(X_2 + 2 \cdot 2X_1) + U$$
$$sX_2 = 2 \cdot 2X_1 - X_2$$
$$Y = X_2 + 2 \cdot 2X_1$$

$$sX_1 = -X_1 + 4X_1 + 2X_2 + 8X_1 + U$$
$$sX_2 = 4X_1 - X_2$$
$$Y = 4X_1 + X_2$$

$$sX_1 = 11X_1 + 2X_2 + U$$
$$sX_2 = 4X_1 - X_2$$
$$Y = 4X_1 + X_2$$

$$\begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} = \begin{bmatrix} 11 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [U]$$

$$[Y] = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + [0][U]$$

$$(s\mathbb{I}-A)^D = \begin{bmatrix} s+1 & 4 \\ 2 & s-11 \end{bmatrix}^T = \begin{bmatrix} s+1 & 2 \\ 4 & s-11 \end{bmatrix}$$

$$\begin{array}{cc|cc|c} & & s+1 & 2 & 1 \\ & & 4 & s-11 & 0 \\ \hline 4 & 1 & 4s+4+4 & 8+s-11 & 4s+8 \end{array}$$

$$H(s) = \frac{4s+8}{s^2-10s-19}$$

$$s^2-10s-19=0$$

$$\Delta_s = 100-4\cdot 19 = 24$$

$$\sqrt{\Delta_s} = 2\sqrt{6}$$

$$s_1 = \frac{10-2\sqrt{6}}{2} = 5-\sqrt{6}$$

$$s_2 = \frac{10+2\sqrt{6}}{2} = 5+\sqrt{6}$$

układ jest stabilny w sensie BIBO
gdy $Re(s_1)<0 \wedge Re(s_2)<0$
a więc układ jest NIE stabilny