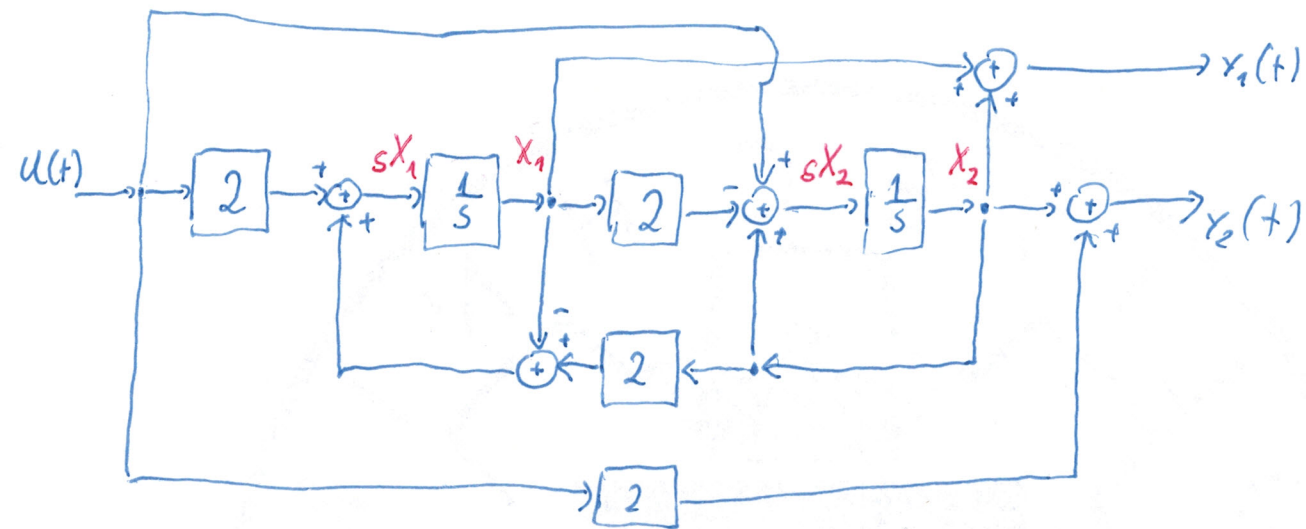


Zadanie 3



$$sX_1 = -X_1 + 2X_2 + 2u$$

$$sX_2 = -2X_1 + X_2 + u$$

$$Y_1 = X_1 + X_2$$

$$Y_2 = X_2 + 2u$$

$$\begin{bmatrix} sX_1 \\ sX_2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} [u]$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} [u]$$

$$A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$H = C \cdot (s\mathbb{I} - A)^{-1} \cdot B + D$$

$$(s\mathbb{I} - A)^{-1} = \frac{(s\mathbb{I} - A)^D}{\det(s\mathbb{I} - A)} = \frac{\begin{bmatrix} s-1 & 2 \\ -2 & s+1 \end{bmatrix}}{s^2+3}$$

$$(s\mathbb{I} - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} s+1 & -2 \\ 2 & s-1 \end{bmatrix}$$

$$\det(s\mathbb{I} - A) = (s+1)(s-1) + 4 = s^2 - 1 + 4 = s^2 + 3$$

$$\begin{array}{c|cc|cc} & s-1 & 2 & 2 & \\ & -2 & s+1 & 1 & \\ \hline \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} & s-1-2 & 2+s+1 & 2s-6+s+3 & 0 \\ & -2 & s+1 & -4+s+1 & 2 \end{array}$$

$$H(s) = \begin{bmatrix} \frac{3s-3}{s^2+3} \\ \frac{s-3}{s^2+3} + 2 \end{bmatrix} = \begin{bmatrix} \frac{3s-3}{s^2+3} \\ \frac{2s^2+s+3}{s^2+3} \end{bmatrix}$$

$$s^2 + 3 = 0$$

$$\Delta = 0 - 4 \cdot 3 = -12$$

$$\sqrt{\Delta} = j\sqrt{4 \cdot 3} = j2\sqrt{3}$$

$$s_1 = \frac{j2\sqrt{3}}{2} = j\sqrt{3}$$

$$s_2 = -j\sqrt{3}$$

układ niestabilny